

LARGE SDSS QUASAR GROUPS AND THEIR STATISTICAL SIGNIFICANCE

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Abstract: We use a volume-limited sample of quasars in the Sloan Digital Sky Survey (SDSS) DR7 quasar catalog to identify quasar groups and address their statistical significance. This quasar sample has a uniform selection function on the sky and nearly a maximum possible contiguous volume that can be drawn from the DR7 catalog. Quasar groups are identified by using the Friend-of-Friend algorithm with a set of fixed comoving linking lengths. We find that the richness distribution of the richest 100 quasar groups or the size distribution of the largest 100 groups are statistically equivalent with those of randomly-distributed points with the same number density and sky coverage when groups are identified with the linking length of $70 h^{-1}\text{Mpc}$. It is shown that the large-scale structures like the huge Large Quasar Group (U1.27) reported by Clowes et al. (2013) can be found with high probability even if quasars have no physical clustering, and does not challenge the initially homogeneous cosmological models. Our results are statistically more reliable than those of Nadathur (2013), where the test was made only for the largest quasar group. It is shown that the linking length should be smaller than $50 h^{-1}\text{Mpc}$ in order for the quasar groups identified in the DR7 catalog not to be dominated by associations of quasars grouped by chance. We present 20 richest quasar groups identified with the linking length of $70 h^{-1}\text{Mpc}$ for further analyses.

Key words: large-scale structure of universe – cosmology: observations – quasars: general

1. INTRODUCTION

The size and mass of large-scale structures ¹ (hereafter LSS) in the universe have been often used as measures of amplitude of density fluctuations on large scales. The CfA Great Wall discovered by de Lapparent et al. (1986) in the CfA2 survey and the Sloan Great Wall in the SDSS survey (Gott et al. 2005) cast doubt on the SCDM model (Geller & Huchra 1989) and the ΛCDM model (Sheth & Diaferio 2011), respectively. It has been argued that the universe did not have enough time for such large structures to form by the present epoch in these cosmological models, and that the gravitational instability theory for structure formation or the Cosmological Principle adopting large-scale homogeneity may not be valid (see also Clowes et al. 2012; Horvath et al. 2013).

On the other hand, a strong support for the Cosmological Principle comes from the smooth cos-

mic microwave background radiation, which shows anisotropies that agrees astonishingly well with those of the initially homogeneous isotropic ΛCDM model (Clifton et al. 2012; Planck collaboration XV 2014). The fluctuation of the galaxy number density on large scales is also found to agree with the prediction of the ΛCDM model (Scrimgeour et al. 2012).

It is expected that LSS of the universe contain a wealth of information on primordial density fluctuations and the history of their growth. This is because they represent density fluctuations on linear scales. It is then very important to rigorously study the physical properties and cosmological meaning of observed superstructures of the universe (Park 1990; Park et al. 2012; Einasto et al. 2011, 2014).

Recently, Clowes et al. (2013) reported a finding of a very large quasar group in the SDSS DR7 quasar catalog (Schneider et al. 2010) with characteristic size of $\sim 350 h^{-1}\text{Mpc}$, longest dimension of $\sim 870 h^{-1}\text{Mpc}$, membership of 73 quasars, and mean redshift $\bar{z} = 1.27$. They claimed that the quasar group, named U1.27, is the largest ‘structure’ currently known in the universe. They argued that their finding might raise a question on validity of the large-scale homogeneity assumption, which is, in the form of Cosmological Principle, one of the fundamental assumptions of the standard cosmology. Their claim falls on the same kind of view applied to CfA and Sloan Great Walls.

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¹ We use the word ‘large-scale structure’ as a general name for structures larger than galaxy clusters. In this terminology the large-scale structures of the universe are classified into high-density and low-density large-scale structures. The high-density large-scale structures are classified into categories of superclusters, filaments, and walls depending on their internal density and morphology. The low-density large-scale structures are voids, and classified into bubbles, and tunnels depending on their morphology. In this scheme all the names that have been used to describe ‘large’ structures in the universe are unified.

However, Nadathur (2013) pointed out that the statistical significance of the huge quasar group has not been properly studied in Clowes et al. (2013), and that Clowes et al.’s claim on the cosmological implication of existence of the large quasar group needs to be examined quantitatively. Nadathur showed that the algorithm used to identify the quasar groups frequently finds such large-size structures even in homogeneous simulations of a Poisson point process with the same number density as the quasar catalog. They concluded that Clowes et al.’s interpretation of U1.27 as a physical ‘structure’ is misleading.

Even though it is cosmologically interesting to search for associations of quasars ² (Webster 1982; Clowes & Campusano 1991; Komberg et al. 1996; Williger et al. 2002; Clowes et al. 2012, 2013; Einasto et al. 2014), one needs to be careful in interpreting the quasar groups identified with specific criteria. This is because the size and richness of LSS depend sensitively on how they are identified. Evolution of the objects used to trace LSS and evolution of LSS themselves can also complicate the interpretation of identified LSS. In particular, the quasar phenomenon has short lifetime and astrophysics of quasar activity can affect the results (Komberg et al. 1996; Wold et al. 2000; Miller et al. 2004; Sochting et al. 2004; Coldwell & Lambas 2006; Hutchings et al. 2009; Lietzen et al. 2009, 2011; Bradshaw et al. 2011; Portinari et al. 2012; Trainor & Steidel 2012; krumpe et al. 2013; Fanidakis et al. 2013; Shen et al. 2013; DiPompeo et al. 2014; Karhunen et al. 2014; Song et al. 2015). Quasar samples usually have very low spatial number density, which can introduce shot noise effects in group identification. This situation demands the use of well-defined samples and rigorous statistical tests.

In this paper we identify quasar groups using a volume-limited sample drawn from the SDSS DR7 quasar catalog that has the maximum contiguous angular size. We present the statistical significance of not only the largest quasar group, but also the quasar groups within a large range of richness. We also address a question whether or not the huge quasar group found by Clowes et al. is a statistically-significant object in terms of multiplicity and size. We find the critical linking length (LL hereafter) for quasar group identification which can result in a catalog dominated by physically clustered groups.

2. SDSS QUASAR CATALOG

We use the fifth edition of the SDSS quasar catalog from SDSS DR7, which is a compilation of quasars probed in the SDSS-I and SDSS-II quasar survey (Schneider et al.

2010). The catalog contains 105783 spectroscopically confirmed quasars in the sky of area covering approximately 9380deg^2 . They are brighter than $M_i = -22.0$ (corrected for the Galactic extinction, and K -corrected to $z = 2$ in a cosmology with $H_0 = 70\text{km/s/Mpc}$, $\Omega_m = 0.3$, and $\Omega_\Lambda = 0.7$) and have at least one broad emission line with full width at half maximum larger than 1000km/s or interesting/complex absorption features. The quasars show a wide redshift distribution from 0.065 to 5.46, but the majority of quasars (76%) is concentrated below redshift 2. They have the apparent i magnitude of $14.86 < i < 22.36$, where the lower limit comes from the maximum brightness limit of the target selection on quasar candidates to avoid saturation and cross-talk in the spectra. The catalog does not include several classes of active galactic nuclei such as Type 2 quasars, Seyfert galaxies, and BL Lacertae objects.

Even though the catalog contains a large number of quasars, not all the quasars are suitable for statistical analyses for several reasons. It is mainly because the quasar target selection for the follow-up spectroscopy was not done uniformly in its early version (see Richards et al. 2006; Shen et al. 2007, for more details). Shen et al. (2011) provided a parameter, in the 10th column in their catalog, named as ‘uniform flag’ to take the uniformity problem into account. The catalog of Schneider et al. also has its own uniform flag in the 35th column. The uniform flag is assigned based on the achievement of the target selection algorithm. But Shen et al. made more careful evaluation of the selection effects by finding out quasars that are uniformly selected as high- z candidates but later identified as low- z quasars. Shen et al. has one more class than Schneider et al. as follows: 0 = not in the uniform sample; 1 = uniformly selected and with the galactic extinction-corrected i magnitude less than 19.1 at $z < 2.9$ or 20.2 at $z > 2.9$; 2 = uniformly selected by the QSO-HiZ branch in the algorithm and with measured spectroscopic redshift $z < 2.9$ and $i > 19.1$. By selecting those whose Shen et al. (2011)’s uniform flag is 1, we construct a uniform sample of 59679 quasars with Galactic extinction corrected i magnitude less than 19.1. It rejects about a half of the quasars in the original catalog, mostly at $z < 2.9$.

In Figure 1 we plot the quasars in the Schneider et al. (2010)’s catalog in black. Only the northern hemisphere of the SDSS survey regions is adopted and the SDSS survey coordinates η and λ are used for the plot as the survey boundaries are more simply defined in this coordinate system. The red points in the figure are quasars marked as those of the uniform subsample by Shen et al.

Since the sky distribution of the uniform quasar sample is not perfectly contiguous, we define a new boundary for our sample by excluding isolated patches and trimming the jagged boundaries to make one clean contiguous region. The green line in Figure 1 defines the region under our study. The area of the region is 1.754 steradian. This is nearly the maximum possible con-

² In this paper we call the large loose groupings of quasars ‘quasar groups.’ This terminology can give a misleading idea that they are compact and physically interacting internally as in galaxy groups. Einasto et al. (2014) used ‘quasar systems’ to avoid the confusion. The name ‘quasar association’ may also be an appropriate name for these loose quasar groups since the name ‘stellar association’ has been already used in astronomy for loose groups of stars that are not gravitationally bound.

tiguous sample can be drawn from the DR7 catalog.

Figure 2 shows the uniform-sample quasars located within our mask in the i -band absolute magnitude and redshift plane. In Figure 3 it can be seen that the comoving space number density of quasars is roughly uniform between $z = 0.8$ and 1.8 , but is significantly higher between $z = 0.1$ and 0.8 .

Song et al. (2015) has compared the distribution of the SDSS quasars at $0.4 \leq z \leq 0.7$ with the smooth density field of the galaxies in the SDSS DR12 BOSS survey and found that the probability of finding a quasar increases monotonically as the local galaxy number density increases with no significant redshift dependence. This implies that LSS found in the distribution of quasars should be similar to those in the galaxy distribution. Then a quasar sample having a constant comoving number density should be as useful as a volume-limited galaxy sample in finding LSS.

In order to make a volume-limited quasar sample with uniform density we apply an absolute magnitude cut line given by

$$M_{i,\text{lim}} = 0.7216z^2 - 3.505z - 22.278 \quad (1)$$

in the range of $0.1 < z < 0.8$. The cut is shown as a blue line in Figure 2. The resulting subsample has a nearly uniform number density of $1.31 \times 10^{-6} (h^{-1}\text{Mpc})^{-3}$ in a wide range of $0.1 < z < 1.8$ as shown by the red histogram in Figure 3. The corresponding mean quasar separation is $91.5 h^{-1}\text{Mpc}$. By varying the absolute magnitude cut one can effectively correct a quasar sample for the luminosity evolution. The use of uniform comoving density sample makes us to sample the same kind of cosmological objects over a wide redshift range.

Our final sample contains 32276 quasars with $i < 19.1$ in the redshift range $0.1 \leq z \leq 1.8$ covering 5758 deg^2 . On the other hand, Clowes et al. (2013) and Nadathur (2013) did not use the sample made to meet the precondition of uniformity for statistical works. They just applied an i -band apparent magnitude cut of $i \leq 19.1$ to Schneider et al. (2010)'s sample, where i here is not corrected for the Galactic extinction. This made their sample affected by non-uniform quasar target selection which Shen et al. (2011) corrected, and the structures identified are affected by the Galactic extinction. Our sample is from the homogeneous sample and has a true flux limit. It is also much larger in the sky area (5758 deg^2 versus 3725 deg^2) and in redshift range ($0.1 \leq z \leq 1.8$ versus $1.0 \leq z \leq 1.8$). It makes our sample contain 1.72 times more quasars than that used by Clowes et al. (2013), Nadathur (2013), and Einasto et al. (2014).

3. IDENTIFICATION OF QUASAR GROUPS

To identify groups of quasars we apply the Friend-of-Friend (FoF) algorithm to the ‘uniform’ constant-number density sample described in the previous section. We first use $LL = 70h^{-1}\text{Mpc}$ for a comparison with Clowes et al. (2013)'s results. This corresponds to the threshold overdensity of $\delta = (\bar{d}/LL)^3 - 1 =$

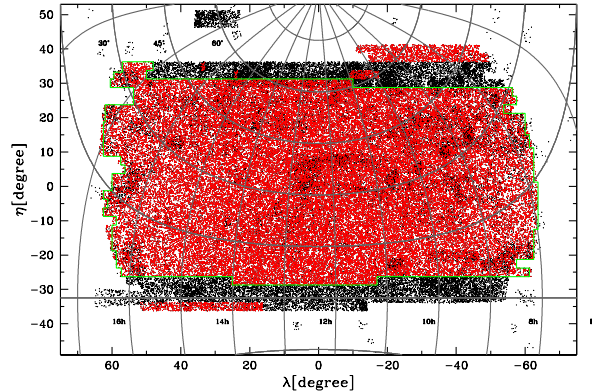


Figure 1. Distribution of all the quasars in the original Schneider et al. (2010)'s catalog (black points) and the quasars in the Shen et al. (2011)'s uniform sample (red points) in the SDSS survey coordinates. A contiguous region defined by the green line is the region under analysis in our study.

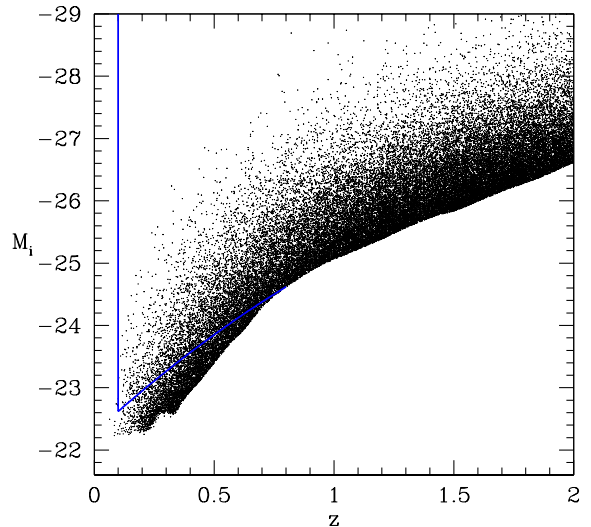


Figure 2. Distribution of the uniform-sample quasars in the redshift versus i -band absolute magnitude plane. The blue line is a low-redshift absolute magnitude cut used in our study making the sample nearly constant in comoving number density between $z = 0.1$ and 1.8 .

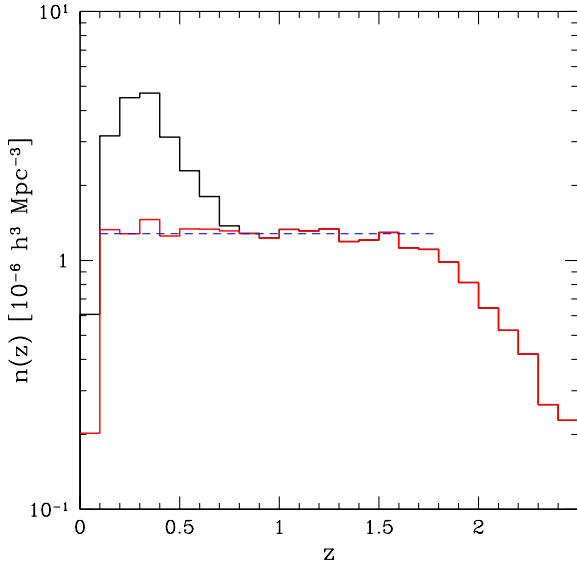


Figure 3. Comoving space number density of quasars. The black line is for the whole uniform sample, and the red line is when the low-redshift absolute magnitude cut (Eq. 1) is applied. The blue dashed line is the mean density between $z = 0.1$ and 1.8 .

$(91.5/70)^3 - 1 = 1.23$, where \bar{d} is the mean quasar separation. We find 3233 quasar groups having 3 or more members. Figure 4 shows the positions of top ten richest groups on the sky (upper panel) and in the x - y plane of Equatorial coordinate frame (bottom panel). The second richest one labelled with ‘2’ has 70 member quasars and the maximum extent of $703 h^{-1}\text{Mpc}$. The maximum extent is defined as the comoving distance between the most distant two members in a group. This group corresponds to huge Large Quasar Group, U1.27, reported by Clowes et al. The group identified by us has 3 new members and does not have 6 members compared to Clowes et al.’s list. We discover a rich quasar group, marked with ‘1’ in Fig. 4, that contains 95 members and the maximum extent of $712 h^{-1}\text{Mpc}$. This group was not found by previous studies because its center is located at $z = 0.366$, which is outside the redshift limit of $1.0 < z < 1.8$ adopted by Clowes et al., Nadathur (2013), and Einasto et al. (2014). This group is much richer and has a size larger than U1.27. A catalog of twenty richest quasar groups is given in Table 1 for future analyses.

To check if our group finding is consistent with previous works we perform the FoF group finding to a sample from the original Schneider et al. (2010)’s catalog that is limited only by $i < 19.1$ not corrected for Galactic extinction and a redshift range of $1.0 < z < 1.8$. In this case the richest group is found to have 73 members that are exactly the same as the member quasars of U1.27 listed in Table 1 of Clowes et al. This proves that the difference in our quasar catalog is not due to the group finding algorithm but to the sample definition. We also compared our quasar groups with those identified by

Table 1

The twenty richest SDSS DR7 quasar groups identified with the linking length of $70 h^{-1}\text{Mpc}$ or overdensity threshold of 1.23.

ID	$\bar{\alpha}(\text{deg})^a$	$\bar{\delta}(\text{deg})^a$	\bar{z}^a	\bar{r}^a	N^b	L^c
1	170.08	29.33	0.366	1012	95	712
2	163.53	14.47	1.244	2757	70	703
3	196.45	39.95	1.112	2546	64	536
4	195.98	27.09	1.554	3198	62	625
5	166.91	33.88	1.075	2485	60	570
6	156.69	38.63	0.580	1519	59	536
7	219.41	27.68	0.598	1560	56	459
8	204.40	13.04	1.215	2712	55	582
9	187.60	43.55	1.404	2995	53	651
10	232.82	23.19	1.511	3141	53	556
11	219.69	19.68	0.764	1909	53	640
12	226.57	16.83	1.062	2463	51	470
13	141.58	47.90	1.157	2620	50	526
14	208.63	25.81	1.236	2745	48	446
15	137.82	21.70	0.957	2279	47	414
16	212.06	28.23	1.080	2493	47	754
17	127.45	20.13	1.351	2917	46	545
18	157.40	22.01	1.493	3117	45	576
19	158.17	15.82	0.744	1869	45	456
20	231.00	47.84	1.508	3138	44	459

The full catalog with information on members of groups can be downloaded at http://astro.kias.re.kr/Quasar/DR7_20groups/.

^a Coordinates of the geometric center of groups. The mean comoving distance \bar{r} is in units of $h^{-1}\text{Mpc}$.

^b The number of member quasars.

^c The maximum extent of groups in $h^{-1}\text{Mpc}$.

Einasto et al. (2014, see their Table A.3) who also used Schneider et al.’s catalog constrained by $i < 19.1$, and found a very good agreement at $z > 1.0$. We choose our sample instead of that used by previous works because the observed sample must be uniformly defined for statistical hypothesis tests against random catalogs. The observed quasar distribution has random perturbations along the line-of-sight due to the random redshift error which is typically about 0.004 in the case of SDSS DR7 catalog (Schneider et al. 2010). Therefore, it is necessary to examine if the quasar group finding is significantly affected by this error. We will discuss it in the next section.

4. COMPARISON WITH RANDOM DISTRIBUTIONS

We generate random point sets which have the same number density and the same angular coverage on the sky as our quasar sample. The random catalogs also have the mean point separation of $91.5 h^{-1}\text{Mpc}$ throughout the entire redshift range from $z = 0.1$ to 1.8 . We make 10000 random mock samples. For each mock sample we find groups of points in the same way as for the quasar sample. We then measure richness and size of the groups in quasar sample and random mock samples. Richness is the number of member quasars/points belonging to a given group. Size is the maximum comoving extent of the group.

In the upper panel of Figure 5 we plot the cumulative distribution function (CDF) of richness. It is the num-

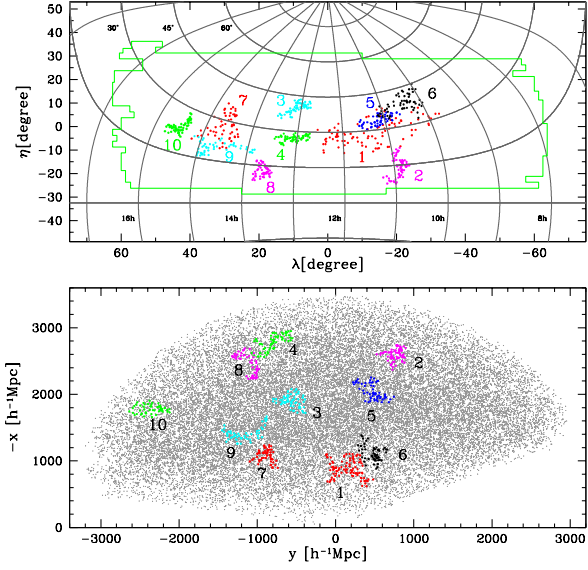


Figure 4. (upper panel) Ten richest quasar groups detected with the linking length of $70h^{-1}$ Mpc. The second richest one in magenta on the bottom right labelled with 2 corresponds to the huge Large Quasar Group, U1.27, of Clowes et al. (2013). (bottom panel) Same as the upper plot but in the Cartesian coordinates defined in the Equatorial coordinate system. Grey points are all the quasars in our sample.

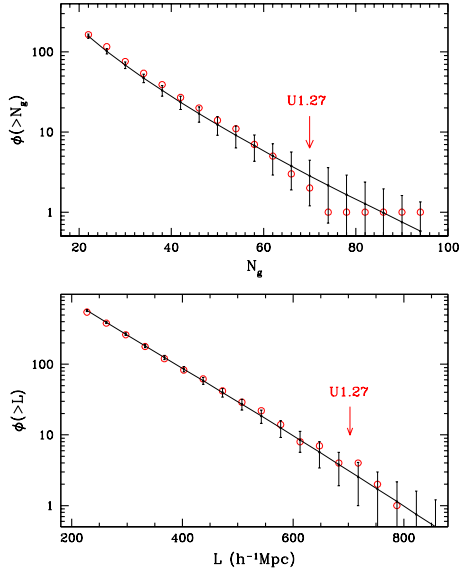


Figure 5. Cumulative distribution functions of richness (upper panel) and maximum extent (bottom panel) of the groups identified in the observed SDSS quasar catalog (red circles) and in random point catalogs (solid line with error bars). A set of 10000 random point catalogs is used for the comparison. The group corresponding to the huge Large Quasar Group, U1.27, reported by Clowes et al. (2013) is marked with downward arrows.

ber of groups whose richness is greater than or equal to N_g . The circles are from the observed quasar sample. Richness of the group corresponding to U1.27 group is 70, and is marked with an arrow. The solid line and error bars are the mean and standard deviations obtained from 10000 random catalogs. Similarly, the bottom panel shows the number of observed quasar groups whose size is greater than or equal to L , and the results from 10000 mock catalogs. We find that U1.27 is $703 h^{-1}$ Mpc long (marked with an arrow) and is only the fourth largest one. It can be seen that the richness and size distributions of the observed quasars are statistically indistinguishable with those of random catalogs.

To estimate the probability of finding groups like the observed quasar groups in random catalogs we perform a χ^2 test. We define

$$\chi^2 = \frac{1}{N} \sum_{i=1}^N (\phi_i - \bar{\phi}_i)^2 / \sigma_i^2 \quad (2)$$

as a measure of the deviation of a particular CDF ϕ from the average CDF $\bar{\phi}$ of random catalogs. Here σ_i is the standard deviation of the CDF's of random catalogs in the i -th bin, and the summation is over the range where the average CDF of random catalogs is between 3 and 100. The choice ensures that a large part of CDF is compared. The histograms in Figure 6 are the distributions of χ^2 of 10000 random catalogs and the arrows correspond to the observed SDSS quasar catalog generated with $LL = 70h^{-1}$ Mpc. The probability of having χ^2 greater than χ_{obs}^2 in random catalogs is 58.1% and 92.4% for richness and size distributions, respectively. Therefore, it can be concluded that the quasar groups identified with $LL = 70h^{-1}$ Mpc are consistent with those of random point distributions.

On the other hand, we find that the probability of finding a random group whose richness is equal to or greater than 70 is 92.3%. Similarly, the probability of finding a random group with the maximum extent equal to or greater than $703 h^{-1}$ Mpc is 100%. Therefore, both richness and size of the objects like U1.27 are of no surprise even in random distributions of points with no physical clustering when LL is as large as $70h^{-1}$ Mpc.

Since quasars do have intrinsic spatial clustering, it is expected that these distribution functions are statistically different from those of random catalogs for some smaller LL or for higher density thresholds. Figure 7 shows the CDF's of richness and size of quasar groups when LL is taken to be 60, 50, and $40 h^{-1}$ Mpc. Also shown are the average CDF's (solid lines with error bars) of random point groups identified with the corresponding LL . It is clear that the CDF's of observed group richness and size become more different from those of random catalogs as LL decreases.

It will be then interesting to search for the critical LL for which the observed CDF becomes significantly different from those of random points. We calculate the probability that the CDF of random point groups happens to show a deviation from the mean CDF as large as or greater than the difference of the observed CDF

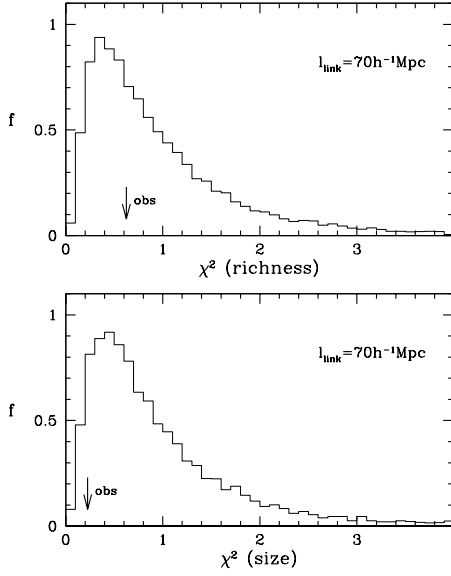


Figure 6. Distributions of χ^2 defined as in Eq. (2) for random point samples. The upper panel is for richness of groups identified with the linking length of $70 h^{-1}\text{Mpc}$, and the bottom panel is for the maximum extent of groups. The χ^2 for the observed sample are marked with arrows.

from the mean. This is estimated by counting the cases with $\chi_{\text{ran}}^2 \geq \chi_{\text{obs}}^2$. A high probability means that the observed groups are statistically consistent with random ones. Figure 8 shows the probability as a function of the LL used to find groups. For both richness and size it can be seen that the observed groups are consistent with those of random catalogs when LL is chosen to be larger than about $50 h^{-1}\text{Mpc}$. Clowes et al. chose $LL = 70 h^{-1}\text{Mpc}$ and it is now clear that the choice makes the group catalog dominated by randomly connected groups. Figures 7 and 8 indicate that the groups identified with $LL < 50 h^{-1}\text{Mpc}$ are likely to be physically-clustered genuine groups of quasars.

As mentioned in the previous section the random redshift error of order of 0.004 seems problematic in finding physical structures as the error corresponds to $\sim 12 h^{-1}\text{Mpc}$. We use the redshifts calculated by Hewett & Wild (2010, hereafter HW10) whose redshift errors are much smaller and redo the group findings. According to the error estimates for the redshifts of HW10, the RMS redshift error for quasars at $0.1 < z < 1.8$ is found to be only 0.00024, more than 10 times smaller than the nominal value for the SDSS DR7.

It will be a good consistency check to compare two group catalogs derived from Schneider et al. (2010) and HW10's redshifts. We find that the new quasar group catalog is very close to the previous one obtained using Schneider et al. (2010)'s redshifts. All major groups are detected again and only a few members are affected. As a result, the richness and size distributions change only slightly. It is reassuring that the quasar groups and the statistics are robust against the redshift error

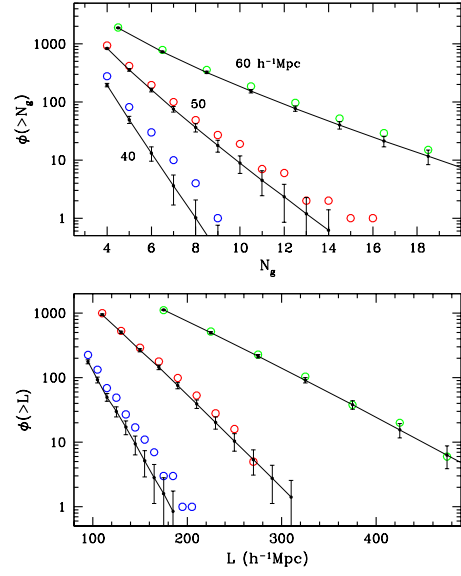


Figure 7. Cumulative distribution functions of richness (upper panel) and size (bottom panel) when the linking length for group identification is taken to be 60, 50, and $40 h^{-1}\text{Mpc}$. Circles are from observation, and solid lines with error bars are from 1000 random catalogs.

of Schneider et al. (2010). When $LL = 50 h^{-1}\text{Mpc}$, we still find a very good agreement of both distribution functions. Therefore, all of our results are essentially unchanged by using more accurate quasar redshifts. We stick to Schneider et al. (2010)'s redshifts in our paper since there is practically no change in the conclusions and it is easier to compare our groups directly to those of previous works. The groups identified in the redshift space are also affected by the peculiar velocities of quasars. Influence of peculiar velocity on group identification is to some extent unavoidable. But this is relatively small because the RMS pairwise peculiar velocity of galaxies is only about 300 km/s .

5. SUMMARY AND CONCLUSIONS

We make a wide contiguous volume-limited sample of quasars statistically uniformly selected from the SDSS DR7 quasar catalog and identify quasar groups using the FoF algorithm. The statistical properties of the observed quasar groups are compared with those of random point sets. We make the following findings.

1. The observed SDSS DR7 quasar groups identified with $LL = 70 h^{-1}\text{Mpc}$ are statistically consistent, in terms of size and richness, with those of random points clustered by chance. The size or richness distribution functions of those quasar groups can be found in random catalogs with 92.4% and 58.1% probabilities, respectively.

2. The probabilities of finding groups larger or richer than the large quasar group U1.27, reported by Clowes et al. (2013), in a random point catalog are 100% and 92%, respectively.

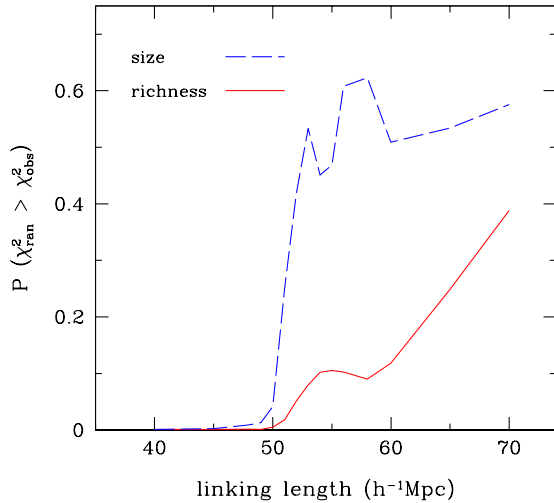


Figure 8. Probability of finding random catalogs whose distribution function deviates from their mean more than or equal to that of the observed sample. The deviation is measured by χ^2 defined in Eq. (2). Probability is measured for richness (solid line) and size (dashed line) distribution functions as a function the linking length.

3. The quasar groups are statistically dominated by physically clustered quasars when LL for group identification is less than $50 h^{-1}\text{Mpc}$.

We emphasize that the observed LSS should not be used to draw cosmological conclusions without making elaborate statistical tests. Size and richness of cosmic structures depend sensitively on how those structures are identified. For example, one can always find structures as large as the survey size if the threshold density level used to identify them is lowered close to the percolation density. Even in a random point distribution without any physical clustering an infinitely long structure can be identified when LL is increased to 0.864 times the mean particle separation, which is the critical value for percolation. It is conceivable that one would claim discovery of groups of galaxies or quasars even larger than the Sloan Great Wall in existing or future redshift surveys. The lesson of the past is that one will again need to make statistical tests before drawing conclusions on cosmological implication of those objects.

Our results suggest that the large quasar groups identified with $LL > 50 h^{-1}\text{Mpc}$ can be considered as associations of smaller physically-clustered quasar systems that are connected by a very large linking length. The smaller quasar systems are similar to the supercluster complexes in the local universe, as was suggested in Einasto et al. (2014). Hutsemékers et al. (2014) showed that quasar spin axes in U1.27 seem to be aligned parallel to this structure. However, from their Figure 6 one can see that alignment is better if this quasar group is divided into smaller systems, in agreement with our interpretation of quasar systems.

There have also been several studies trying to define and measure the homogeneity scale of the universe (Yadav et al. 2010; Marinoni et al. 2012; Scrimgeour et al. 2012). However, definition of the homogeneity scale has been either arbitrary or dependent on observational samples. We point out that, in the current popular cosmological models that adopt the Cosmological Principle, the notion of the critical scale of homogeneity does not make sense. Homogeneity is only achieved asymptotically in these models as the scale of observation increases, and there is no characteristic scale intrinsic to the universe above which the universe can be suddenly regarded homogeneous (see also Nadathur 2013). Instead of looking for arbitrarily-defined homogeneity scales or naively examining the size of the largest structures the observed LSS should be statistically compared with the structures identified in the same way in the mock survey samples obtained from simulated initially-homogeneous universes. In our forthcoming paper we will use mock quasars formed in simulated universes to identify large groups of quasars and statistically compared their properties with those of observed quasars. We will examine if the large structures can be explained in the framework of the standard hierarchical structure formation model in the given Hubble time.

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